Adjustment with least squares method
Two software packages – Two results

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Adjustment with least squares method

Several software packages use the "least squares method" for adjustment of observations to calculate the coordinates. The mathematical background of this method is to minimize the weighted sum of squares, $v^T P v \rightarrow \min$ consequently the evaluation of this sum is a possibility to compare the results of different programs and their algorithms.

Based on a simulated network the true coordinates are known.

$$X_0^T = [\mathbf{x}_1 \quad \mathbf{y}_1 \quad \mathbf{z}_1 \quad . \quad . \quad . \quad \mathbf{x}_n \quad \mathbf{y}_n \quad \mathbf{z}_n]$$

The true observations are calculated from the true coordinates.

$$\tilde{L}^T = [\mathbf{r}_1 \quad . \quad \mathbf{r}_n \quad \mathbf{z}_1 \quad . \quad \mathbf{z}_n \quad \mathbf{d}_1 \quad . \quad \mathbf{d}_n]$$

The standard deviation of azimuth and zenith is set to $\sigma_{r/z}$ =0.3mgon, the standard deviation of distance is set to σ_d =0.05mm+0ppm.

The observation vector L consists of the true observations $\tilde{L}$ and a random error vector ε , which is scaled with the standard deviation of each obervation.

$$L = \tilde{L} + \varepsilon$$

$$\varepsilon_i = t_i \times \sigma_{r/z/d}$$

$$\varepsilon^T = [\boldsymbol{\varepsilon}_{r1} \quad . \quad \boldsymbol{\varepsilon}_{rn} \quad \boldsymbol{\varepsilon}_{z1} \quad . \quad \boldsymbol{\varepsilon}_{zn} \quad \boldsymbol{\varepsilon}_{d1} \quad . \quad \boldsymbol{\varepsilon}_{dn}]$$

The Gaussian variables t_i were randomized with the Box-Muller-Method. Such numbers can be numerically sampled from two uniform ([0, 1]) random numbers u_1 and u_2 through the formula

$$t = \cos(2 * \pi * u_1) * \sqrt{(-2 * \ln u_2)}$$

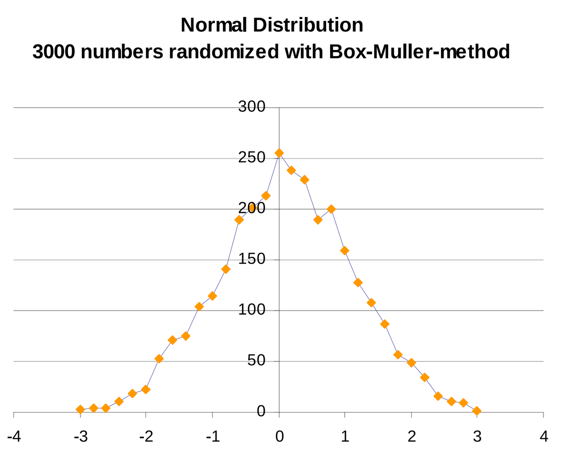

Figure 1

Now the generated observations can be adjusted considering the least squares method

$$v^T P v \rightarrow \min$$

The adjusted observations $\hat{L}$ are the sum of the observations L and the correction v.

$$\hat{L} = L + v$$

In ideal case the correction v should be equal to the random error ε

Design of a simulated 3-D-network

Development

A 3D-network with 10 points and three instruments was simulated, covering an area of 30 * 30 m² with elevation difference up to 4m. Because of the fact, that in SA an instrument stand cannot be a measured point, the instruments have the same theoretical coordinates as the identical target points but a different name.
For all network simulations the observation data is related to gravity.

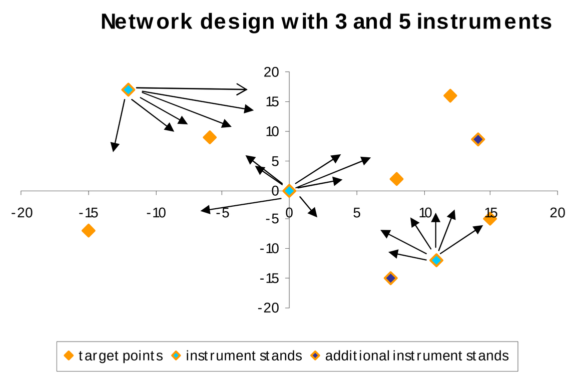

Figure 2

Calculation

The 'true' observations (distance, azimuth, zenit) for the three instruments were calculated.

These 81 observations were distorted by the described method and imported in PANDA and Spatial Analyzer. The true point-coordinates were imported as coarse coordinates to locate the instruments in SA and for further calculations in the software packages.

The standard deviations were set to 0.3mgon and 0.05mm+0ppm.
PANDA runs a so called 'free adjustment' no point is fixed, all points together define the datum of the network.

In SA a so called 'Unified Spatial Metrology Network' is calculated, rotations around x and y are disabled for all instruments. The adjusted coordinates are saved in a new point group and the instruments are transformed in SA.

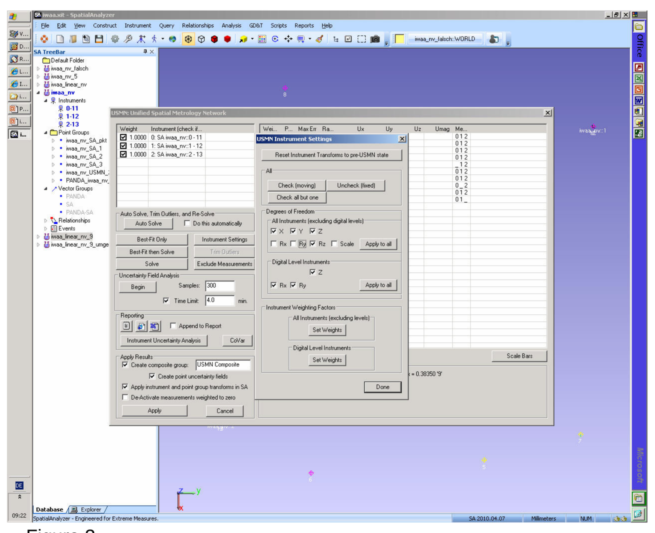

Figure 3

Adjustment

The output of PANDA includes the adjusted coordinates and observations, also the correction of the observations and the sum of squares of the corrections.

SA represents the adjusted coordinates of the measured points in a point group, the coordinates of the instruments are shown in the instrument properties.

The differences to the true coordinates are shown in figure 3.

Vectors of correction: Magnification 100000 PANDA black SA red

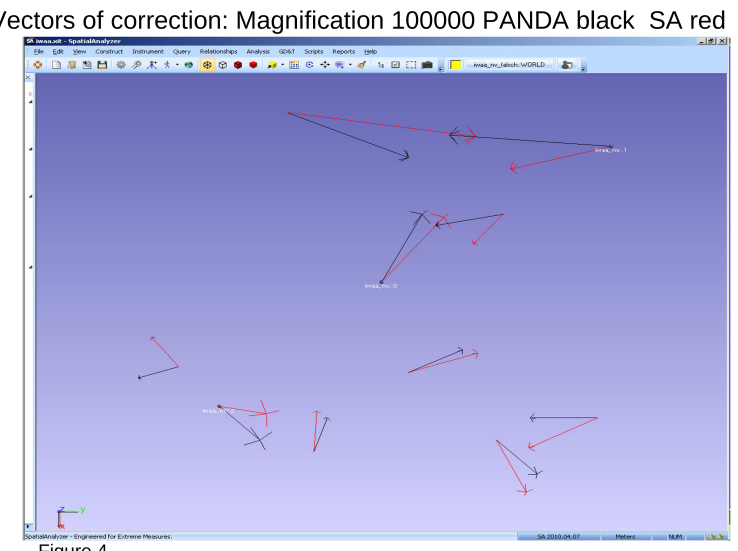

Figure 4

Result

The adjusted coordinates are used to calculate the adjusted observations in order to get the corrections and their sum of squares.

This step is not really necessary in PANDA, but was done to get a feeling for the accuracy of calculation.

The sums of squares are shown in the following table:

	distance	azimuth	zenith	Sum of squares
PANDA	6,24610	18,10920	27,56550	51,92080
PANDA calc.	6,29878	17,93526	27,62089	51,85494
SA –USMN calc.	6,20966	29,14863	27,35207	62,71036
SA-USMN with point	7,11357	26,73884	28,87664	62,72905

Table 1

As shown in the table for this network a 'Unified Spatial Metrology Network with point group' was calculated,too. The orignal coordinates were allowed to move.

There are little variances between USMN with and without point groups in the individual sum of squares, but finally the total sum of squares doesn't change.

Upgrading the 3D-network to 5 instruments

Because of the good corresponding results in this network with three instruments, two more instruments (dark blue in figure 2) were integrated and the number of observations rises up to 135. These observations were also distorted, but with new random numbers, and again the observations were adjusted. In the figure below the vectors of correction are shown:

Vectors of correction: Magnification 100000 PANDA black SA red

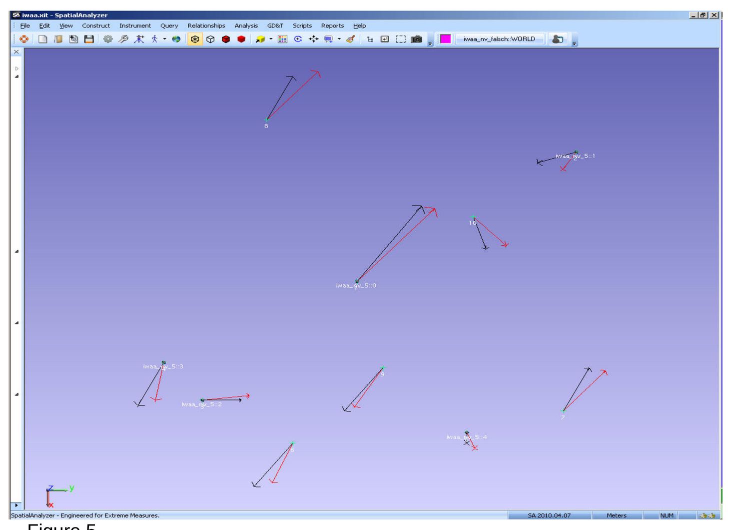

Figure 5

Sum of squares

	distance	azimuth	zenith	Sum of squares
PANDA	27,02930	20,36210	33,87020	81,26160
PANDA calc.	28,26602	20,53047	34,01268	82,80916
SA –USMN calc.	28,00942	28,95287	33,60847	90,57076

Table 2

Two more instrument stands and 54 more observations have of course an effect on the individual sum of squares for distance and zenith, but not really for the azimuth. In the azimuth sum of squares of Spatial Analyzer there is even a small reduction.

Summary

Surprisingly the sum of squares of Spatial Analyzer and PANDA are very similar concerning the distance and zenith angle, but for the azimuth Spatial Analyzer gives a much larger value than PANDA.

Therefore, from the analytical point of view the Spatial Analyzer result is suboptimal.

Design of a 3-D Linear-Accelerator-Network

Network configuration

Additionally, a second network has been used for comparison.
It has been simulated as a linear accelerator network, which is similar to the ones commonly used at DESY.

Four points, left and right of the theoretical beamline, two of them on the bottom, the other on the ceiling, build a 'ring'. Every ten meters another ring is installed. Overall there are nine rings. The instrument is placed in the middle of two rings and observes usually two rings forward and two rings backward.

In this example seven instruments are used, so that the last ring is measured without redundancy.

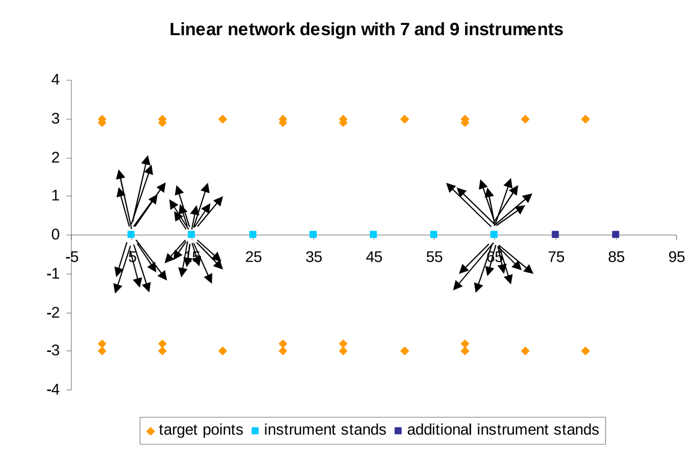

Figure 6

Calculation

The 'true' observations (distance, azimuth, zenith) for the seven instruments and 36 measured points were calculated. The 384 observations were distorted and imported in PANDA and Spatial Analyzer.

PANDA adjusted the 384 observations and didn't show corrections for the 12 observations to the four points of the last ring, which are only observed once (as said before).

SA adjusted only the 372 observations to the points which are at least observed twice, but surprisingly the 12 unchecked observations also changed. At the moment there is no explanation for this behaviour.

Result

Vectors of correction: Magnification 100000 PANDA black SA red

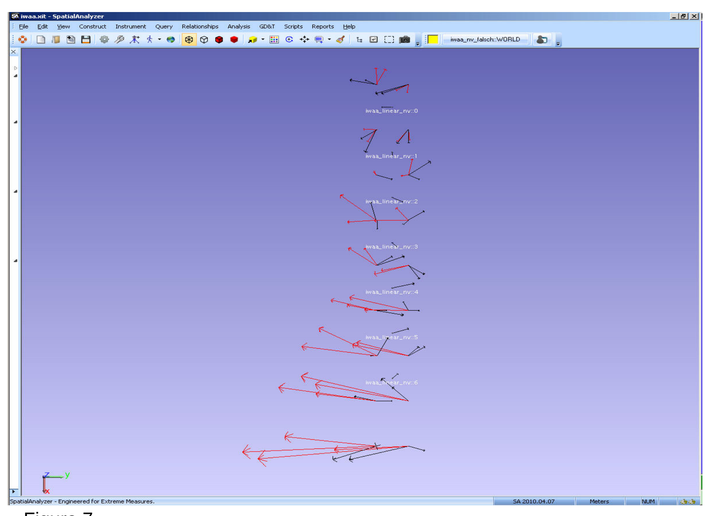

Figure 7

While for the PANDA solution the distribution of correction looks relatively consistent, for the SA result a lateral movement of the adjusted coordinates is pronounced. A short look at the sum of squares for azimuths confirms that the SA-solution is 1.5 times higher than the PANDA result for the azimuths.

In this network the individual sum of squares for distance and Zenith are equal for both results within the limits of accuracy of calculation.

However, the total sum of squares is significantly larger for SA than for PANDA.

	distance	azimuth	zenith	Sum of squares
PANDA	98,68740	76,24530	85,48060	260,41330
PANDA calc.	96,48888	76,18897	85,26244	257,94028
SA –USMN calc.	97,07441	122,50218	85,44770	305,02429

Table 3

Redundancy

To answer the question whether the difference at the end of the network can be explained with the unused observations, the network was extended with two additional instruments so that now the last ring is also redundant.

Now nine instruments are used, the number of (new) distorted observations is 468, while the number of target points is still 36.

Vectors of correction: Magnification 100000 PANDA black SA red

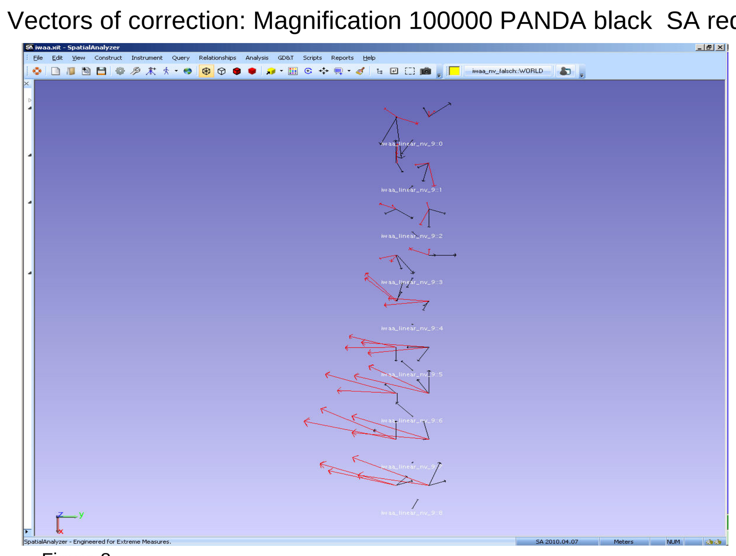

Figure 8

	distance	azimuth	zenith	Sum of squares
PANDA	117,83290	94,58250	104,68590	317,10130
PANDA calc.	117,07540	94,89817	104,82915	316,80271
SA –USMN calc.	134,63800	122,82293	122,49341	379,95434

Table 4

The result of this linear network is again surprising: the sum of squares for azimuths of the Spatial Analyzer solution doesn't really change, but now there are also differences to the PANDA adjustment in the other sums of squares, which didn't exist in the other networks.

Measurement setup

Last but not least: It is particularly remarkable that the SA vectors of correction grow from top to bottom or from instrument 1 to instrument 9 whereas the size of the PANDA vectors of correction is consistent in this net.

So, does the solution in SA depend on the internal order of the instruments?
There are two possibilities to check this idea.

First one is to copy the instruments in reverse order in a new collection and adjust the network again. There is no difference in the results for this approach.

The second possibility is to create an new collection and read in the instruments and point groups (measurements) in reverse order. The adjustment of this data shows a mirrored result for the vectors of correction.

That means that the timestamp of the measurements is a critical information for the adjustment of Spatial Analyzer.

Vectors of correction: Magnification 100000 SA red

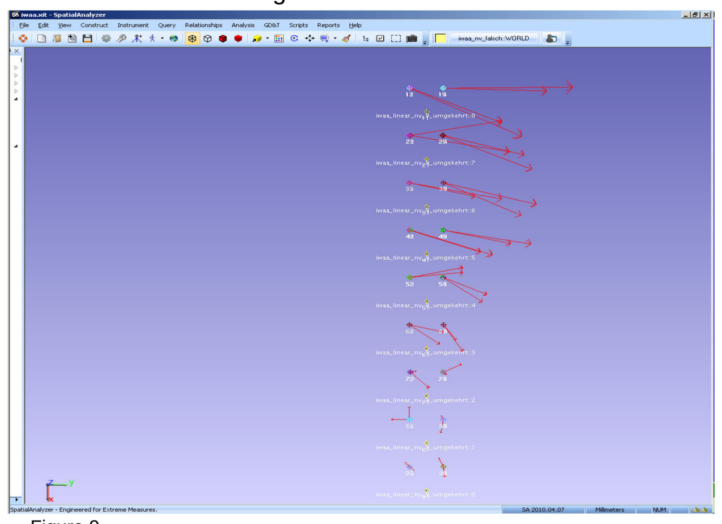

Figure 9

Conclusion

A method for the comparison of different network adjustment packages has been shown. So far only PANDA and SA have been evaluated with a limited set of simulated networks.
While both packages claim to use something like $v^T P v \rightarrow \min$, with the least square method developed by Gauss, there are significant differences in the results.

With the comparison of the sum of squares obviously the rightness of a solution can not be proven. So at the moment the minimal sum of squares has to be considered the right one. In all estimated networks the weighted sum of squares of the residuals was larger for the SA solution than for PANDA.

Especially the lateral movement of points from start to end is important for the acclerator community. This behaviour ist dependent on the internal order of observations and maybe caused by a sequential estimation of network points in SA.
It is planned to analyse more network configurations in the future.
People from other laboratories with different software packages are welcome to contribute.

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ASSOCIATION